Formulae On Three Dimensional Geometry

Equation of a line in space

- Equation of a *line passing through a point A with position vector* $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ *and parallel to a given vector* $\vec{b} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ *where P be any point on the line with position vector* $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by:
  - **Vector form:** $\boldsymbol{\vec{r} = \vec{a} + \lambda\vec{b}}$
  - **Cartesian form:** $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ $where\ a, b, c\ are\ direction\ ratios$

    OR $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ $where\ l, m, n\ are\ direction\ cosines$

- Equation of a *line passing through the points* $A\ (x_1, y_1, z_1)$ *and* $B\ (x_2, y_2, z_2)$ *with position vectors* $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ *and* $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ *where P be any point on the line with position vector* $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by:
  - **Vector form:** $\boldsymbol{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$
  - **Cartesian form:** $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Angle between two lines

- If $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ be the vector form of any two lines, then angle between them is given by: $\boldsymbol{\cos\theta} = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$
- If $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_1}{a_2} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_2}$ be the cartesian forms of two lines, then the angle between them is given by: $\boldsymbol{\cos\theta} = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

Shortest distance between two skew lines

- If $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ be the vector forms of any 2 skew-lines, then the shortest distance between them is given by:

$$\boldsymbol{d} = \left|\frac{(\boldsymbol{b_1} \times \boldsymbol{b_2}).(\boldsymbol{a_2} - \boldsymbol{a_1})}{|\boldsymbol{b_1} \times \boldsymbol{b_2}|}\right|$$

- If $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_1}{a_2} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_2}$ be the cartesian forms of any 2 skew-lines, then the shortest distance between them is given by:

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

Shortest distance between two parallel lines

- If $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ be the vector forms of any 2 parallel lines, then the shortest distance between them is given by:

$$\boldsymbol{d} = \left|\frac{\vec{\boldsymbol{b}} \times (\vec{\boldsymbol{a_2}} - \vec{\boldsymbol{a_1}})}{|\vec{\boldsymbol{b}}|}\right|$$

Equation of a plane

Equation of a plane in normal form:

- If $\hat{n}$ is the unit normal vector to a plane whose distance from the origin is d and $P(x, y, z)$ is any point on the plane with position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then the equation of the plane is given by:
  - **Vector form:** $\vec{\boldsymbol{r}}.\hat{\boldsymbol{n}} = \boldsymbol{d}$
  - **Cartesian form:** $\boldsymbol{lx} + \boldsymbol{my} + \boldsymbol{nz} = \boldsymbol{d},$ *where* l, m, n *are the direction cosines of* $\hat{n}$

Equation of a plane perpendicular to a given vector and passing through a given point:

- If a plane passes through a point $A(x_1, y_1, z_1)$ with position vector $\vec{a}$ and is perpendicular to the vector $\vec{N}$ where $P(x, y, z)$ is any point on the plane with position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then the equation of the plane is given by:
  - **Vector form:** $(\vec{r} - \vec{a}).\vec{N} = 0$
  - **Cartesian form:** $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

Equation of a plane passing through three non-collinear points:

- If $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be any three points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and $P(x, y, z)$ is any point on the plane with position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then the equation of the plane is given by:
  - **Vector form**: $(\vec{r} - \vec{a}) \times \left[\left(\vec{b} - \vec{a}\right) \times (\vec{c} - \vec{a})\right] = 0$
  - **Cartesian form**: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

Intercept form of the equation of a plane:

- If the equation of a plane is $Ax + By + Cz + D = 0$ making intercepts a, b & c on X, Y & Z- axes, then intercept form of the equation of the plane is:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Plane passing through the intersection of two given planes:

- **Vector form:** If $\vec{r}.\widehat{n_1} = d_1$ and $\vec{r}.\widehat{n_2} = d_2$ are two planes intersecting each other, then equation of the plane passing through the line of intersection is given by:

$$\vec{r}.(\vec{n}_1 + \lambda\overrightarrow{n_2}) = d_1 + \lambda d_2$$

- **Cartesian Form:** If $\overrightarrow{n_1} = A_1\hat{\imath} + B_1\hat{\jmath} + C_1\hat{k}$ and $\overrightarrow{n_2} = A_2\hat{\imath} + B_2\hat{\jmath} + C_2\hat{k}$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then the required equation of the plane is

$$(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$$

Coplanarity of Two Lines

- **Vector Form:** If $\vec{r} = \vec{a_1} + \boldsymbol{\lambda}\vec{b}_1$ and $\vec{r} = \vec{a_2} + \boldsymbol{\lambda}\vec{b}_2$ are two lines then they are coplanar if and only if
  - $\overrightarrow{\boldsymbol{AB}}.\left(\overrightarrow{\boldsymbol{b_1}} \times \overrightarrow{\boldsymbol{b_2}}\right) = \mathbf{0}$ [Where $\overrightarrow{AB} = \vec{a_2} - \vec{a_1}$] *OR*
  - $(\overrightarrow{\boldsymbol{a_2}} - \overrightarrow{\boldsymbol{a_1}}).\left(\overrightarrow{\boldsymbol{b_1}} \times \overrightarrow{\boldsymbol{b_2}}\right) = \mathbf{0}$

- **Cartesian Form:** $\begin{vmatrix} \boldsymbol{x_2 - x_1} & \boldsymbol{y_2 - y_1} & \boldsymbol{z_2 - z_1} \\ \boldsymbol{a_1} & \boldsymbol{b_1} & \boldsymbol{c_1} \\ \boldsymbol{a_2} & \boldsymbol{b_2} & \boldsymbol{c_2} \end{vmatrix} = \mathbf{0}$

Distance of a point from a plane

- **Vector Form:** If P is a point with position vector $\vec{a}$ and a plane with a equation $\vec{r}.\vec{N} = d$, where $\widehat{N}$ is the normal to the plane then the distance between them is:

$$\frac{|\vec{\boldsymbol{a}}.\vec{\boldsymbol{N}} - \boldsymbol{d}|}{|\vec{\boldsymbol{N}}|}$$

- **Cartesian Form:** If $P(x_1, y_1, z_1)$ be a point with a position vector $\vec{a}$ and $Ax + By + Cz = D$ is the equation of a plane and $\vec{N} = A\hat{\imath} + B\hat{\jmath} + C\hat{k}$ is the normal to the plane, then distance between them is given by:

$$\left|\frac{\boldsymbol{Ax_1 + By_1 + Cz_1 - D}}{\sqrt{\boldsymbol{A^2 + B^2 + C^2}}}\right|$$

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