BROAD ROOT SCHOLARS ACADEMY

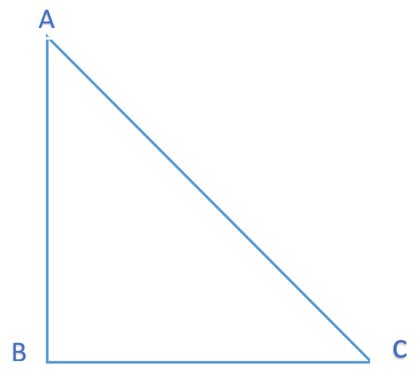
General Formulae on Trigonometry

Sexagesimal System (Degree Measure)		
$1^0 = 60'$	$1' = 60''$	
Centasimal System (French System)		
$90^0 = 100\ grad = 100^g$	$1^g = 100'$	$1' = 100''$
Circular System (Radian Measure)		
$\pi\ radian\ (\pi^c) = 180^0$	$1^c = \left(\frac{180}{\pi}\right)^0 = 57^0 16' 22''$	$1^0 = \left(\frac{\pi}{180}\right)^c = 0.01746\ rad$

- ❖ The length $'l'$ of an arc subtending an angle θ at the cente of a circle with radius ‘r’ is given by $\boldsymbol{l = r\ \theta}$
- ❖ The area of a sector of circle of radius ‘r’ bounded by an arc with an angle θ is given by $\boldsymbol{\frac{1}{2} r^2 \theta}$

TRIGONOMETRIC RATIOS

- $\sin A = \frac{perpencidular}{hypotenus} = \frac{AB}{AC}$
- $\cos A = \frac{base}{hypotenus} = \frac{BC}{AC}$
- $\tan A = \frac{perpendicula}{base} = \frac{AB}{BC}$
- $Cosec\ A = \frac{hypotenus}{perpendicular} = \frac{AC}{AB}$
- $\sec A = \frac{hypotenus}{base} = \frac{AC}{BC}$
- $\cot A = \frac{base}{perpendicular} = \frac{BC}{AB}$



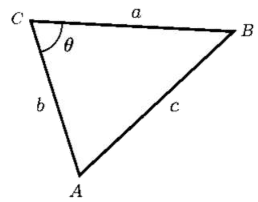
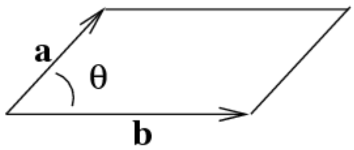
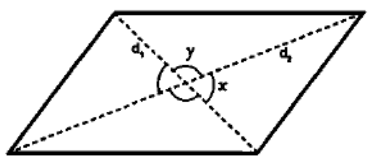
TRIGONOMETRIC RECIPROCAL FORMULA

$cosec\ A = \frac{1}{\sin A}$	$\sec A = \frac{1}{\cos A}$	$\cot A = \frac{1}{\tan A}$	$\tan x = \frac{\sin x}{\cos x}$	$\tan x = \frac{\sin x}{\cos x}$

TRIGONOMETRIC RATIOS FOR STANDARD ANGLES

	0^0 *or* 0^c	30^0 *or* $\left(\frac{\pi}{6}\right)^c$	45^0 *or* $\left(\frac{\pi}{4}\right)^c$	60^0 *or* $\left(\frac{\pi}{3}\right)^c$	90^0 *or* $\left(\frac{\pi}{2}\right)^c$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

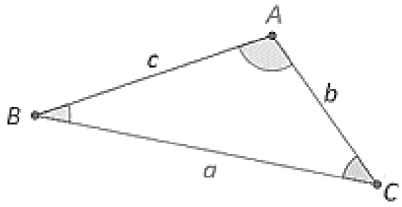
AREAS USING TRIGONOMETRY

Triangle	$$A = \frac{1}{2}ab\sin C$$ a and b are two adjacent sides of a triangle and C is the included angle between the sides	
Parallelogram	$$A = ab\sin\theta$$ a and b are two adjacent sides of a triangle and C is the included angle between the sides	
Quadrilateral	$$A = \frac{1}{2} \times d_1 \times d_2 \times \sin x$$ d_1 and d_2 are the diagonals and x is the angle between the diagonals.	

SINE RULE

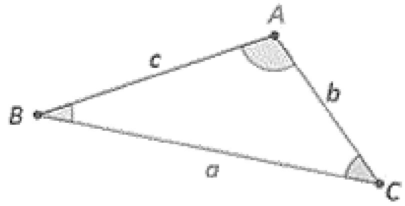
In a ΔABC, where a, b, c are the lengths of the sides, then:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



COSINE RULE

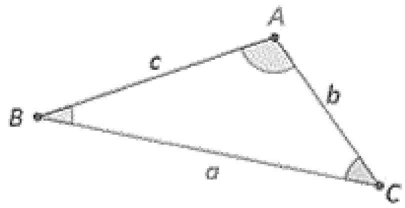
In a ΔABC, where a, b, c are the lengths of the sides, then:



- $\cos A = \frac{b^2+c^2-a^2}{2bc}$
- $\cos B = \frac{a^2+c^2-b^2}{2ac}$
- $\cos C = \frac{a^2+b^2-c^2}{2ab}$

PROJECTION RULE

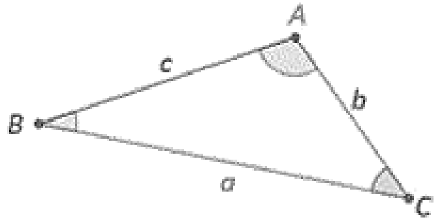
- $a = b\cos C + c\cos B$
- $b = a\cos C + c\cos A$
- $c = a\cos B + b\cos A$



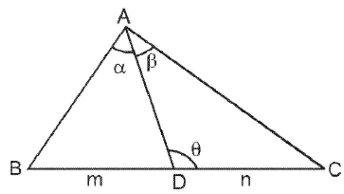
TANGENT RULE/NAPIER'S ANALOGY

- $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$
- $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$
- $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$

TRIGONOMETRIC FUNCTIONS RELATED TO TRIANGLES

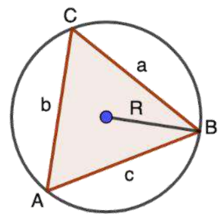


- $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
- $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$
- $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$, $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$
- $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$
- ***$m - n$ rule***:

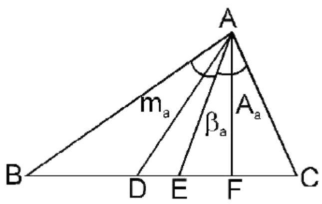


If $BC : DC = m : n, then$

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta = n\cot B - m\cot C$$



- Radius of circumcircle: $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$
- Length of angle bisector, median and altitude:



✓ Length of an angle bisector from the angle A= $\beta_a = \frac{2bc\cos\frac{A}{2}}{b+c}$

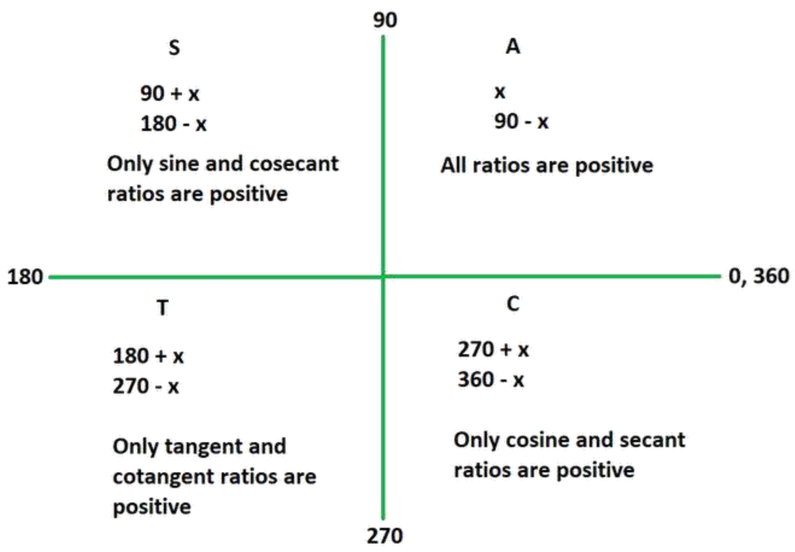
✓ Length of median from the angle A= $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

✓ Length of altitude from the angle A= $A_a = \frac{2\ Area\ of\ triangle}{a}$

CIRCUALR FUNCTIONS

For the even multiple of 90^0, a trigonometric ratio remains same and the sign of the ratio will be assigned according to its respective quadrant. However, for the odd multiple of 90^0, a trigonometric ratio changes as follows:

$$\sin \leftrightarrow cos, \quad tan \leftrightarrow cot, \quad sec \leftrightarrow cosec$$



Trigonometric ratios of $90^0 - \theta$

Since, n=1, it is an odd multiple of 90^0, hence ratio will change. 1st quadrant, All positive.

- $\sin(90^0 - \theta) = \cos\theta$
- $\cos(90^0 - \theta) = \sin\theta$
- $\tan(90^0 - \theta) = \cot\theta$
- $\cot(90^0 - \theta) = \tan\theta$
- $\sec(90^0 - \theta) = \operatorname{cosec}\theta$
- $\operatorname{cosec}(90^0 - \theta) = \sec\theta$

Trigonometric ratios of $90^0 + \theta$

Since, n=1, it is an odd multiple of 90^0, hence ratio will change. 2nd quadrant, only functions of **sin** and **cosec** are positive.

- $\sin(90^0 + \theta) = \cos\theta$
- $\cos(90^0 + \theta) = -\sin\theta$
- $\tan(90^0 + \theta) = -\cot\theta$
- $\cot(90^0 + \theta) = -\tan\theta$
- $\sec(90^0 + \theta) = -\operatorname{cosec}\theta$
- $\operatorname{cosec}(90^0 + \theta) = \sec\theta$

Trigonometric ratios of $180^0 - \theta$

Since, n=2, it is an even multiple of 90^0, hence ratio will not change. 2nd quadrant, only functions of **sin** and **cosec** are positive.

- $\sin(180^0 - \theta) = \sin\theta$
- $\cos(180^0 - \theta) = -\cos\theta$
- $\tan(180^0 - \theta) = -\tan\theta$
- $\cot(180^0 - \theta) = -\cot\theta$
- $\sec(180^0 - \theta) = -\sec\theta$
- $\operatorname{cosec}(180^0 - \theta) = \operatorname{cosec}\theta$

Trigonometric ratios of $180^0 + \theta$

Since, n=2, it is an even multiple of 90^0, hence ratio will not change. 3rd quadrant, only functions of **tan** and **cot** are positive.

- $\sin(180^0 + \theta) = -\sin\theta$
- $\cos(180^0 + \theta) = -\cos\theta$
- $\tan(180^0 + \theta) = \tan\theta$
- $\cot(180^0 + \theta) = \cot\theta$
- $\sec(180^0 + \theta) = -\sec\theta$
- $\operatorname{cosec}(180^0 + \theta) = -\operatorname{cosec}\theta$

Trigonometric ratios of $270^0 - \theta$

Since, n=3, it is an odd multiple of 90^0, hence ratio will change. 3rd quadrant, only functions of **tan** and **cot** positive.

- $\sin(270^0 - \theta) = -\cos\theta$
- $\cos(270^0 - \theta) = -\sin\theta$
- $\tan(270^0 - \theta) = \cot\theta$
- $\cot(270^0 - \theta) = \tan\theta$
- $\sec(270^0 - \theta) = -\operatorname{cosec}\theta$
- $\operatorname{cosec}(270^0 - \theta) = -\sec\theta$

Trigonometric ratios of $270^0 + \theta$

Since, n=3, it is an odd multiple of 90^0, hence ratio will change. 4th quadrant, only functions of **cos** and **sec** are positive.

- $\sin(270^0 + \theta) = -\cos\theta$
- $\cos(270^0 + \theta) = \sin\theta$
- $\tan(270^0 + \theta) = -\cot\theta$
- $\cot(270^0 + \theta) = -\tan\theta$
- $\sec(270^0 + \theta) = \operatorname{cosec}\theta$
- $\operatorname{cosec}(270^0 + \theta) = -\sec\theta$

Trigonometric ratios of $360^0 - \theta$
Since, n=4, it is an even multiple of 90^0, hence ratio will not change. 4^{th} quadrant, only functions of **cos** and **sec** are positive.
• $\sin(360^0 - \theta) = -\sin\theta$ • $\cos(360^0 - \theta) = \cos\theta$ • $\tan(360^0 - \theta) = -\tan\theta$ • $\cot(360^0 - \theta) = -\cot\theta$ • $\sec(360^0 - \theta) = \sec\theta$ • $\text{cosec}(360^0 - \theta) = -\text{cosec}\,\theta$

Trigonometric ratios of $360^0 + \theta$
Since, n=4, it is an even multiple of 90^0, hence ratio will not change. 1^{st} quadrant again, all are positive.
• $\sin(360^0 + \theta) = \sin\theta$ • $\cos(360^0 + \theta) = \cos\theta$ • $\tan(360^0 + \theta) = \tan\theta$ • $\cot(360^0 + \theta) = \cot\theta$ • $\sec(360^0 + \theta) = \sec\theta$ • $\text{cosec}(360^0 + \theta) = \text{cosec}\,\theta$

- 90^0 & 270^0 can be written as 1×90^0 & 3×90^0, hence these are considered as an odd multiple of 90^0.
- 180^0 & 360^0 can be written as 2×90^0 & 4×90^0, hence these are considered as an even multiple of 90^0

TRIGONOMETRIC IDENTITIES

- $sin^2A + cos\,^2A = 1$
- $1 + tan^2A = sec^2A$
- $1 + cot^2A = cosec^2A$

TRIGONOMETRIC FORMULAE ON MULTIPLE ANGLE

- $\sin 2x = 2\sin x\, cos\ x = \frac{2\tan x}{1+tan^2x}$
- $\cos 2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x = \frac{1-tan^2x}{1+tan^2x}$
- $\tan 2x = \frac{2\tan x}{1-tan^2x}$
- $\sin 3x = 3\sin x - 4sin^3x$
- $\cos 3x = 4cos^3x - 3\cos x$
- $\tan 3x = \frac{3\tan x - tan^3x}{1-3tan^2x}$

TRIGONOMETRIC FORMULAE ON COMPOUND ANGLE

- $\sin(x+y) = \sin x \cos y + \cos x \sin\ y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin\ y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$
- $\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$
- $\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$
- $\cos(x+y) - \cos(x-y) = -2\sin x \sin y$
- $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$
- $\sin(x+y) - \sin(x-y) = 2\cos x \sin y$

INVERSE TRIGONOMETRIC RECIPROCAL FORMULAS

- $sin^{-1}x = cosec^{-1}\frac{1}{x}$
- $cos^{-1}x = sec^{-1}\frac{1}{x}$
- $tan^{-1}x = cot^{-1}\frac{1}{x},\ x > 0 = cot^{-1}\frac{1}{x} - \pi, x < 0$
- $cot^{-1}x = tan^{-1}\frac{1}{x},\ x > 0 = tan^{-1}\frac{1}{x} + \pi, x < 0$
- $cosec^{-1}x = sin^{-1}\frac{1}{x}$
- $sec^{-1}x = cos^{-1}\frac{1}{x}$

INVERSE TRIGONOMETRIC FORMULAS FOR NEGATIVE ANGLES

- $sin^{-1}(-x) = -sin^{-1}x$
- $tan^{-1}(-x) = -tan^{-1}x$
- $cosec^{-1}(-x) = -cosec^{-1}x$
- $cos^{-1}(-x) = \pi - cos^{-1}x$
- $sec^{-1}(-x) = \pi - sec^{-1}x$
- $cot^{-1}(-x) = \pi - cot^{-1}x$

INVERSE TRIGONOMETRIC IDENTITIES

- $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$
- $tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$
- $cosec^{-1}x + sec^{-1}x = \frac{\pi}{2}$

INVERSE TRIGONOMETRIC FORMULAE ON COMPOUND ANGLE

- $sin^{-1}x + sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$
- $sin^{-1}x - sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$
- $cos^{-1}x + cos^{-1}y = cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$
- $cos^{-1}x - cos^{-1}y = cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$
- $tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}$
- $tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1+xy}$

INVERSE TRIGONOMETRIC FORMULAE ON MULTIPLE ANGLE

- $2sin^{-1}x = sin^{-1}\left(2x\sqrt{1-x^2}\right)$
- $2cos^{-1}x = cos^{-1}(2x^2 - 1)$
- $2tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right) = sin^{-1}\frac{2x}{1+x^2} = cos^{-1}\frac{1-x^2}{1+x^2}$
- $3sin^{-1}x = sin^{-1}(3x - 4x^3)$
- $3cos^{-1}x = cos^{-1}(4x^3 - 3x)$
- $3tan^{-1}x = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

COMPLEX & HYPERBOLIC TRIGONOMETRIC FUNCTIONS

- $e^{i\theta} = \cos\theta + i\sin\theta$
- $e^{\theta} = \cos h\theta + \sin h\theta$
- $e^{-\theta} = \cos h\theta - \sin h\theta$
- $\sin ix = i\sin hx$
- $cosex\ ix = -i\ cosec\ hx$
- $\cos ix = \cos hx$
- $\sec ix = \sec hx$
- $\tan ix = i\tan hx$
- $\cot ix = -i\cot hx$

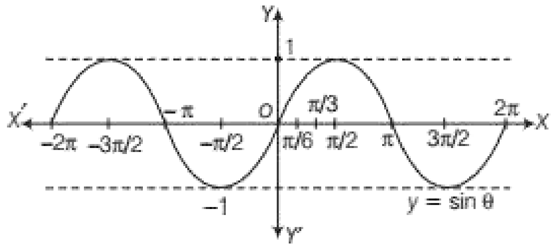
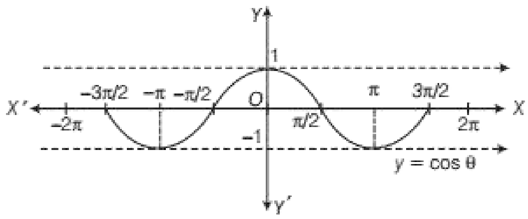
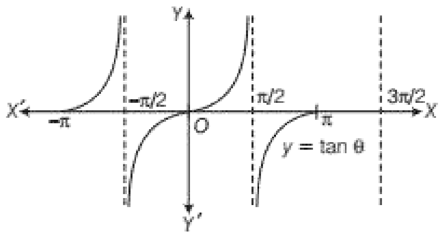
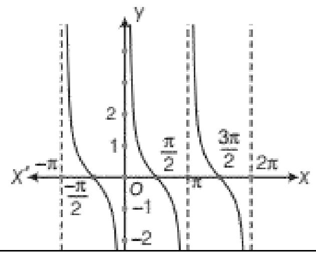
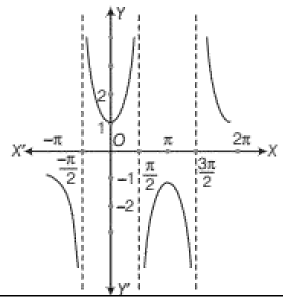
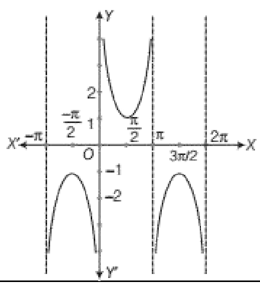
DOMAIN & RANGE OF TRIGONOMETRIC FUNCTIONS

Functions	Domain	Range
$\sin x$	R	$[-1,1]$
$\cos x$	R	$[-1,1]$
$\tan x$	$R-\left\{\frac{(2n+1)\pi}{2}, n\epsilon Z\right\}$	R
$\cot x$	$R-\{n\pi, n\epsilon Z\}$	R
$\sec x$	$R-\left\{\frac{(2n+1)\pi}{2}, n\epsilon Z\right\}$	$R-[-1,1]$
$\operatorname{cosec} x$	$R-\{n\pi, n\epsilon Z\}$	$R-[-1,1]$

GENERAL SOLUTIONS OF TRIGONOMETRIC FUNCTIONS

- If $\sin x = 0$ then $\mathbf{x = n\pi}$, where $n\epsilon Z$
- If $\cos x = 0$ then $\mathbf{x = (2n+1)\frac{\pi}{2}}$, where $n\epsilon Z$
- If $\sin x = \sin y$ then $\mathbf{x = n\pi + (-1)^n y}$, where $n \in Z$
- If $\cos x = \cos y$ then $\mathbf{x = 2n\pi \pm y}$, where $n \in Z$
- If $\tan x = \tan y$ then $\mathbf{x = n\pi + y}$, where $n \in Z$

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Function	Graph
sin x	
cos x	
tan x	
cot x	
sec x	
cosec x	

BROAD ROOT SCHOLARS ACADEMY

FROM BASICS TO ADVANCE.....

COMPLETE GUIDANCE FOR BOARD + NEET/JEE

[OLYMPIAD | CUET-UG | IPMAT | JIPMAT | IISC | IISER | NISER | NDA]

FOUNDATION PROGRAME

[Class VIII-X]

- GENERAL MATHEMATICS
- ADVANCED MATHEMATICS
- GENERAL SCIENCE
- Extended/Core MATHEMATICS [Cambridge IGCSE]

INTEGRATED PROGRAME

[Class XI-XII]

- PHYSICS
- CHEMISTRY
- MATHEMATICS [Core/Applied]
- BIOLOGY

HOW LEARNING HAPPENS HERE

- CONCEPT **CLARITY** FIRST
- REGULAR **TESTS + FEEDBACK**
- **DOUBT CLEARING** SESSIONS
- **PERSONAL ATTENTION** TO EVERY STUDENT
- **NO ROTE** LEARNING METHODS

RESULTS ARE A BY-PRODUCT OF THE PROCESS

IS THIS COACHING CENTRE RIGHT FOR YOUR CHILD?

DOES YOUR CHILD

- **STRUGGLE** WITH CONCEPT?
- NEED **PERSONAL ATTENTION?**
- **AFRAID** OF MATHS & SCIENCE?
- WANT **STRONG FUNDAMENTALS?**
- HATE **ROTE LEARNING?**

IF **YES**, THEN YOU'RE IN THE RIGHT PLACE.

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